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A New Type of Skew Uniform Distribution

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ABSTRACT
We introduce a skewed version of the continuous uniform distribution based on a general procedure to extend distributions proposed by Cortés et al. [Journal of Probability and Statistics, Vol. 2018, pp. 1-10.]. We determine the density and the cumulative distribution function of the new distribution and study diverse properties like moments, moment generating function, shape characteristics, quantile function, Hazard rate, entropy and maximum likelihood estimators. We apply the new model to describe COVID-19 mortality rates and online product reviews.

KEYWORDS
Skewed distributions; distribution theory; Hazard rate function; Shannon entropy; COVID-19 death rates; product reviews

1. Introduction

In the last few decades, many efforts have been made to transform or extend known probability distributions in order to make them more flexible for applications. While classical models may be too "rigid" to be fitted to real world data, the extended distributions aim to accommodate specific features like skewness, bimodality or fat tails, for example. One of the oldest approaches to obtain skew distributions consists of defining a density by

$$h(x) = 2f(x)G(\lambda x),$$

where f is a probability density (pdf), G a cumulative distribution function (cdf) and λ a new parameter introduced to control skewness and kurtosis. Usually, f and G refer to random variables that are symmetric about 0. This approach has been pursued by many authors. In particular, Abid (2015), Chang et al. (2005, Section 3), Nadarajah and Aryal (2004) and Nadarajah and Kotz (2005, Section 7) have used this idea to define a skewed version of the continuous uniform distribution. Somewhat more elaborated attempts to create skew distributions are the alpha-skew generalized distributions, see Elal-Olivero (2009, p. 226), Shams Harandi and Almatsaz (2013,

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p. 775), Acitas et al. (2015, p. 356) and the alpha-beta-skew generalized models, see Esmaeili et al. (2019, p. 607). A more general method to extend distributions, which incorporates the aforementioned procedures, was developed by Cortés et al. (2018).

In the present paper we apply the latter procedure to the uniform distribution. The results may be useful for practice, since the uniform distribution appears in many areas like random number generation, physics, epidemiology, finance, traffic flow modeling and hypothesis testing. The distribution presented here may also serve as an alternative to other distributions supported on a bounded interval like e.g. the Beta or the Kumaraswamy distribution.

In the next section, we present the density, the cumulative distribution function and moments of the general skew uniform distribution. In Section 3, we focus on the standard form of the skew uniform distribution, studying pdf, cdf, Hazard rate function, moments, shape characteristics, quantile function and the moment generating function. Section 4 studies the Shannon entropy and maximum likelihood equations of the new probability model. Finally, Section 5 presents applications in modeling COVID-19 mortality rates and product reviews.

2. The new distribution

Cortés et al. (2018, p.2, eq. (3)) proposed a general construction principle to obtain a new continuous probability distribution. Let $g(x)$ be a given density and $h(x)$ a positive continuous function such that $K = \int h(x)g(x)dx$ is finite. It can be easily shown that

$$f(x) = \frac{1 + \xi h(x)}{1 + K\xi}\, g(x) \tag{1}$$

is a probability density with shape parameter $\xi \geq 0$. The constant K can be interpreted as the expectation of a random variable X with density $g(x)$. In the present article we assume that

$$g(x) = \frac{1}{d - c} \quad \text{for } c \leq x \leq d \tag{2}$$

is a uniform distribution and the positive function h is given by

$$h(x) = (1 - ax - bx^3)^2. \tag{3}$$

We obtain the general skew uniform density as

$$f(x) = \frac{1 + \xi(1 - ax - bx^3)^2}{1 + K\xi}\, \frac{1}{d - c} \tag{4}$$

where K is defined as above. Setting

$$C = (1 + K\xi)(d - c) \tag{5}$$

we can write the density as

$$f(x) = \frac{1+\xi(1-ax-bx^3)^2}{C} \quad \text{for } c \le x \le d \tag{6}$$

which is a polynomial of degree 6 in x, and C is the (inverse) normalizing constant.

After expanding the numerator in (6) one can calculate the corresponding cumulative distribution function as

$$F(x) = \int_c^x f(s)ds =$$

$$\frac{\xi b^2(x^7-c^7)}{7C} + \frac{2\xi ab(x^5-c^4)}{5C} - \frac{\xi b(x^4-c^4)}{2C} + \frac{\xi a^2(x^3-c^3)}{3C} - \frac{\xi a(x^2-c^2)}{C} + \frac{(1+\xi)(x-c)}{C}. \tag{7}$$

Some algebraic manipulations yield an analogous expression for the noncentral moments:

$$\begin{aligned}\mu_k' = E(X^k) = {} & \frac{\xi b^2(d^{k+7}-c^{k+7})}{(k+7)C} + \frac{2\xi ab(d^{k+5}-c^{k+5})}{(k+5)C} - \frac{2\xi b(d^{k+4}-c^{k+4})}{(k+4)C} \\ & + \frac{\xi a^2(d^{k+3}-c^{k+3})}{(k+3)C} - \frac{2\xi a(d^{k+2}-c^{k+2})}{(k+2)C} + \frac{(1+\xi)(d^{k+1}-c^{k+1})}{(k+1)C}.\end{aligned} \tag{8}$$

The latter can be written in the compact form

$$\mu_k' = \frac{1}{C}\sum_{i=1}^{7} r_i \frac{d^{k+i}-c^{k+i}}{k+i}, \tag{9}$$

where the coefficients r_i are given as

$$(r_1, \ldots, r_7) = (1+\xi, -2\xi a, \xi a^2, -2\xi b, 2\xi ab, 0, \xi b^2). \tag{10}$$

Using (9), the expectation and the variance are obtained as

$$E(X) = \mu_1' = \frac{1}{C}\sum_{i=1}^{7} r_i \frac{d^{1+i}-c^{1+i}}{1+i} \tag{11}$$

and

$$V(X) = \mu_2' - (\mu_1')^2 = \frac{1}{C}\sum_{i=1}^{7} r_i \frac{d^{2+i}-c^{2+i}}{2+i} - \frac{1}{C^2}\left(\sum_{i=1}^{7} r_i \frac{d^{1+i}-c^{1+i}}{1+i}\right)^2. \tag{12}$$

There are no "simple" closed forms for the central moments, but by means of an adequate software for symbolic calculation, e.g. MAPLE, one can easily calculate them by means of the formula $\mu_k = E((X-\mu_1')^k)$. The same holds for shape characteristics based on moments, like skewness and kurtosis. We will return to this topic in the next section.

3. Standard form of the skew uniform distribution

In order to reduce complexity, we limit ourselves from now on to the uniform distribution over $[0,1]$, i.e. we assume that in (2.2) it holds $c = 0$ and $d = 1$.

From (6) we obtain the density

$$g(y) = \frac{1 + \xi(1 - ay - by^3)^2}{C_0} \quad \text{for } 0 \le y \le 1, \tag{13}$$

where

$$C_0 = 1 + \xi\left(\frac{b^2}{7} + \frac{2ab}{5} - \frac{b}{2} + \frac{a^2}{3} - a + 1\right).$$

The distribution (13) will be called the *abξ-skew uniform distribution (SU)*. If the parameter values tend to infinity, the following limiting distributions arise:

$$\lim_{\xi\to\infty} g(y) = \frac{1 + \xi(1 - ay - by^3)^2}{\frac{b^2}{7} + \frac{2ab}{5} - \frac{b}{2} + \frac{a^2}{3} - a + 1}, \tag{14}$$

$$\lim_{a\to\pm\infty} g(y) = 3y^2, \quad \lim_{b\to\pm\infty} g(y) = 7y^6.$$

Note that if Y has a skew uniform distribution over $[0,1]$, then

$$X = (d - c)Y + c \tag{15}$$

has a skew uniform distribution over the general interval $[c,d]$.

From (7) we obtain the cdf as

$$G(y) = \frac{1}{C_0}\left(\frac{\xi b^2 y^7}{7} + \frac{2\xi ab y^5}{5} - \frac{\xi b y^4}{2} + \frac{\xi a^2 y^3}{3} - \xi a y^2 + (1+\xi)y\right). \tag{16}$$

Fig. 1 illustrates examples for the pdf and the cdf of the distribution in question. It turns out that the density may assume very different shapes with one to three modes. Note that the derivative of (13) is

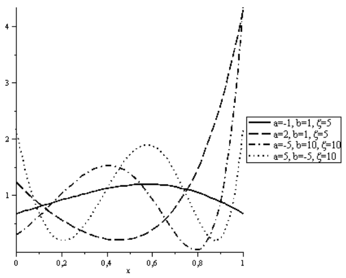
$$g'(y) = -\frac{2\xi}{C_0}(1 - ay - by^3)(a + 3by^2). \tag{17}$$

An elementary analysis shows that (17) has at most three zeros and the density (13) can have at most three modes. For $\xi = 0$ or $a = b = 0$ the constant density $g(y) = 1$ over $[0,1]$ is obtained.

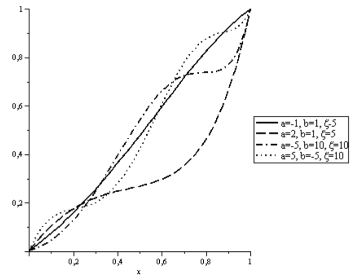
From (13) and (16) one can easily obtain the Hazard rate function as

$$R(y) = \frac{g(y)}{1 - G(y)} = \frac{1 + \xi(1 - ay - by^3)^2}{C_0 - y - \xi\left(\frac{b^2y^7}{7} + \frac{2aby^5}{5} - \frac{by^4}{2} + \frac{a^2y^3}{3} - ay^2 + y\right)} \tag{18}$$

which is an important quantity of survival analysis and may be interpreted as the rate of death (failure) of an item that has reached a certain age. Some examples of the



(a) Density given by (13).



(b) Cumulative distribution function given by (16). Each cdf corresponds to the fdp in part (a) with the same linestyle.

Figure 1. Density and cumulative distribution function of the $ab\xi$-skew uniform distribution.

function (18) are illustrated in Fig. 2. The selected parameter values correspond to those in Fig. 1.

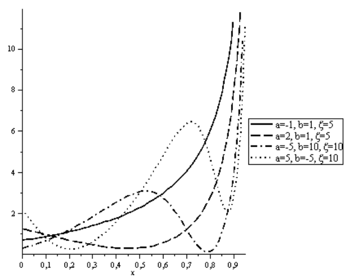


Figure 2. Hazard rate function (18) of the $ab\xi$-skew uniform distribution.

We obtain the noncentral moments from (8) as

$$\mu'_k = \frac{1}{C_0}\left(\frac{\xi b^2}{k+7} + \frac{2\xi ab}{k+5} - \frac{2\xi b}{k+4} + \frac{\xi a^2}{k+3} - \frac{2\xi a}{k+2} + \frac{1+\xi}{k+1}\right). \tag{19}$$

In particular we get the expectation

$$E(X) = \mu'_1 = \frac{1}{C_0}\left(\frac{\xi b^2}{8} + \frac{2\xi ab}{6} - \frac{2\xi b}{5} + \frac{\xi a^2}{4} - \frac{2\xi a}{3} + \frac{1+\xi}{2}\right). \tag{20}$$

Using e.g. MAPLE one can derive formulas for the central moments $\mu_k = E((X-\mu'_1)^k)$ and from these one obtains formulas for the skewness $\gamma_1 = \mu_3/\mu_2^{3/2}$ and the kurtosis

$\gamma_2 = \mu_4/\mu_2^2$. For example, in the case $\xi = 1$, $b = 0$ the model (13) reduces to

$$g(y) = \frac{1 + (1 - ay)^2}{C_0} = \frac{1 + (1 - axy)}{\frac{a^2}{3} - a + 2} \tag{21}$$

This is the alpha skew uniform distribution, see e.g Elal-Olivero (2009, p. 226, eq. (2.2)), Shams and Alamatsatz (2013) and Acitas et al. (2015) for analogous concepts. The distribution (21) has the following characteristics

$$E(Y) = \mu_1' = \frac{3a^2 - 8a + 12}{4a^2 - 12a + 24} \tag{22}$$

$$V(Y) = \mu_2 = \frac{3a^4 - 24a^3 + 128a^2 - 240a + 240}{80(a^2 - 3a + 6)^2} \tag{23}$$

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{a(a^5 - 12a^4 + 96a^3 - 296a^2 + 432a - 288)\sqrt{20}}{-(3a^4 - 24a^3 + 128a^2 - 240a + 240)^{3/2}} \tag{24}$$

$$\gamma_2 = \frac{\mu_4}{\mu_2^2} = \frac{15}{7}\,\frac{13a^8 - 208a^7 + 204a^6 - 10400a^5 + 33120a^4 - 70656a^3 + 104448a^2 - 96768a + 48384}{(3a^4 - 24a^3 + 128a^2 - 240a + 240)^2} \tag{25}$$

which are graphically illustrated in Figure 3.

It might be interesting to observe that the above measures may have several local minima and maxima. For the specific case $\xi = 1, b = 0$ in (13) considered above (see (21)) one can also obtain a closed-form representation of the quantile function, i.e. the inverse of the cdf (16). To this end the equation

$$G(y) = \frac{1}{C_0}\left(\frac{a^2y^3}{3} - ay^2 + 2y\right) = p \tag{26}$$

has to be solved for y, where $0 \le p \le 1$ and $C_0 = \frac{a^2}{3} - a + 2$. There are two complex conjugate solutions and one real solution y_0, given by

$$y_0 = \frac{L - 1/L + 1}{a}$$

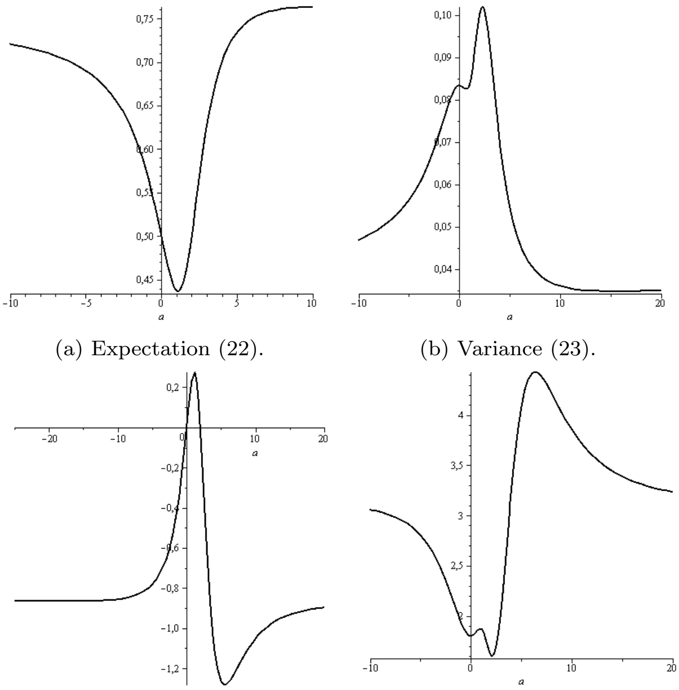
where

$$L = \left(-2 + \frac{3}{2}pC_0a + \sqrt{5 - 6pC_0a + \frac{9}{4}(pC_0a)^2}\right)^{1/3}. \tag{27}$$

The quantile function is therefore

$$Q(p) = G^{-1}(p) = \frac{L - 1/L + 1}{a} \tag{28}$$

with L as in (27). Using the Inverse Transform Method one can now easily simulate outcomes of the alpha-skew uniform distribution (21). It is only necessary to construct a sample $(u_1, \ldots, u_n)$ of the uniform distribution over $[0, 1]$. Then a sample of (21) is



(a) Expectation (22). (b) Variance (23).

(c) Skewness (24). (d) Kurtosis (25).

Figure 3. Characteristics of the alpha-skew distribution (21).

given by $(Q(u_1), \ldots, Q(u_n))$. Fig. 4 illustrates the histogram of 10000 outcomes of the distribution (21), generated in this way. The solid line is the corresponding density. It turns out that the density corresponds well to the stochastic representation of the random variable.

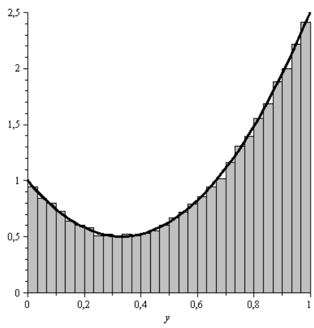


Figure 4. Simulation of the alpha-skew distribution (21) for $a = 3$.

In principle, the Inverse Transform Method works also for the general cdf (17). In this case the values $Q(\mu_1), \ldots, Q(\mu_n)$ must be determined by a numerical procedure, for example, the Newton-Raphson method to solve the equation $G(y) = u_i$ for y $(i = 1, \ldots, n)$.

Using integration by parts, the moment generating function of the -skew uniform distribution (13) can be written as

$$M_Y(t) = \int_0^1 e^{ty} g(y)dy = \frac{1}{C_0 t^7}\left(p_1(t)e^t + p_2(t)\right), \tag{29}$$

where $p_1(t)$ and $p_2(t)$ are polynomials defined by

$$p_1 = (1 + (a-1+b)^2\xi)t^6 - 2\xi(a+3b)(a-1+b)t^5 + 2(15b^2 + (12a-6)b + a^2)\xi t^4$$
$$- 48\left(\frac{5b}{2} - \frac{1}{4} + a\right)b\xi t^3 + 48\left(\frac{15b}{2} + a\right)b\xi t^2 - 720\xi b^2 t + 720\xi b^2$$

and

$$p_2 = -(1+\xi)t^6 - 2\xi a t^5 - 2a^2\xi t^4 - 12b\xi t^3 - 48ab\xi t^2 - 720\xi b^2.$$

The characteristic function is obtained by substituting t in (29) by it, where $i = \sqrt{-1}$. Since X is a linear transformation of Y (see (15)), the moment generating function of X is given by

$$M_X(t) = e^{tc} M_Y((d-c)t) = \frac{e^{tc}}{C_0(d-c)^7 t^7}\left[p_1((d-c)t)e^{(d-c)t} + p_2((d-c)t)\right].$$

4. Entropy and Inference

The Shannon entropy of a continuous distribution with pdf $g(y)$ is defined as $H = -\int_0^1 g(y)\ln(g(y))dy$. It appears that no closed-form representation exists for the entropy, when $g(y)$ is given by (13). We restrict ourselves again to the alpha skew uniform distribution with density of the form (21).

By means of the substitution $z = 1 - ay$ and several algebraic manipulations we obtain the explicit expression for the entropy:

$$H(a) = -\int_0^1 3\,\frac{1+(1-ay)^2}{a^2-3a+6}\ln\left(3\,\frac{1+(1-ay)^2}{a^2-3a+6}\right)dy$$
$$= \frac{3a(-a^2+3a-6)\ln\left(3\,\frac{a^2-2a+2}{a^2-3a+6}\right) + 12\left(\ln(a^2/2 - a + 1) - \arctan(a-1)\right) - 3\pi + 2a(a^2-3a+9)}{3a(a^2-3a+6)}.$$

The dependence of the entropy from the parameter a is illustrated in Fig.5.

The entropy takes the global maximum at $a = 0$. Another local maximum occurs at $a = 1.751$, while two local minima occur at $a = 1.180$ and $a = 12.39$. Furthermore, it holds that $\lim_{a\to\pm\infty} H(a) = \frac{2}{3} - \ln(3) = -0.4319$. Since the Shannon entropy aims to quantify the amount of uncertainty of a distribution, it might be interesting to look for an interpretation of the local extrema.

For a given sample $(y_1, \ldots, y_n)$ the likelihood function of the $ab\xi$-skew uniform

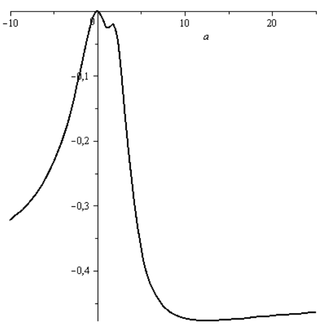


Figure 5. Shannon entropy of the alpha-skew distribution (21).

distribution (13) is given by

$$L = \prod_{i=1}^{n} g(y_i) = \frac{1}{C_0^n} \prod_{i=1}^{n} \left(1 + \xi(1 - ay_i - by_i^3)^2\right).$$

By taking the logarithm one obtains the log-likelihood function as

$$l = \sum_{i=1}^{n} \ln\left(1 + \xi(1 - ay_i - by_i^3)^2\right) - n\ln(C_0).$$

The partial derivatives of the function l are

$$\frac{\partial l}{\partial \xi} = \sum_{i=1}^{n} \frac{(1 - ay_i - by_i^3)^2}{1 + \xi(1 - ay_i - by_i^3)^2} - \frac{n}{C_0}\left(\frac{b^2}{7} + \frac{2ab}{5} - \frac{b}{2} + \frac{a^3}{3} - 1\right),$$

$$\frac{\partial l}{\partial a} = \sum_{i=1}^{n} \frac{-2\xi(1 - ay_i - by_i^3)^2 y_i}{1 + \xi(1 - ay_i - by_i^3)^2} - \frac{n\xi}{C_0}\left(\frac{2b}{5} + \frac{2a}{3} - 1\right),$$

$$\frac{\partial l}{\partial b} = \sum_{i=1}^{n} \frac{-2\xi(1 - ay_i - by_i^3)^2 y_i^3}{1 + \xi(1 - ay_i - by_i^3)^2} - \frac{n\xi}{C_0}\left(\frac{2b}{7} + \frac{2a}{5} - \frac{1}{2}\right).$$

The likelihood equations are now obtained by setting these derivatives to zero. By means of the notation

$$E_i = \frac{(1 - ay_i - by_i^3)^2}{1 + \xi(1 - ay_i - by_i^3)^2}$$

the likelihood equations can be written in the compact form

$$\sum_{i=1}^{n} E_i = \frac{n}{C_0}\left(\frac{b^2}{7} + \frac{2ab}{5} - \frac{b}{2} + \frac{a^3}{3} - 1\right),$$

$$-2\xi \sum_{i=1}^{n} E_i y_i = \frac{n\xi}{C_0}\left(\frac{2b}{5} + \frac{2a}{3} - 1\right),$$

$$-2\xi \sum_{i=1}^{n} E_i y_i^3 = \frac{n\xi}{C_0}\left(\frac{2b}{7} + \frac{2a}{5} - \frac{1}{2}\right).$$

5. Applications

As Fig. 1 illustrates, the $ab\xi$-skew uniform distribution has a flexible density which may be constant (e.g. for $\xi = 0$), convex, concave or even oscillating. The density can have up to five local extremes (see the dotted curve in Fig. 1). Interesting applications arise in the case of U-shaped or similar (J-shaped) data. U-shaped distributions play an important role in modeling disease incidences (COVID-19, tuberculosis) and mortality rates of persons in dependence of age, see e.g. Khera et al. (2021) and Chaimovicz (2001). They are also used in consumptivity studies, in particular in modeling online product reviews, where ratings from 1 to 5 or 1 to 10 are common. It is interesting to observe that consumers tend to avoid medium ratings. They usually participate in a review only, when they "love or hate" the product, which results in U-shaped distributions, see Askay (2013), Hu et al. (2007) and Venkatesakumar et al. (2020). Furthermore, the mentioned distributions also arise in the study of failure rates. In the following we present two numerical examples for the fitting of the skew uniform distribution to practical data. We compare its performance with that of other distributions over $[0, 1]$.

5.1. World wide COVID-19 death rates

We apply the $ab\xi$-skew uniform distribution to the COVID-19 death rates of persons in dependence of their age. Due to Khera et al. (2021), the risk of dying from COVID-19 decreases during childhood up to the age of 3 to 10 years, depending on the country where the study was performed. The worldwide average of the minimum values is about 8 years. After reaching this minimum, the risk of death increases exponentially. In Fig. 6 the logarithmic worldwide death rates are fitted by the skew uniform distribution, some generalized beta and the Kumaraswamy distributions.

The histogram indicates the observed mortality. The solid and the dashed lines indicate the fitted curves of the general skew distribution (13) and its limiting case for $\xi = \infty$, see (14). The dash-dotted and the dotted line represent McDonald's generalized beta distribution and Libby and Novick's generalized beta distribution, respectively. It turns out that the classical beta distribution, McDonald's generalization and the Kumaraswamy distribution are visually indistinguishable from each other in Fig. 6.

In Tab. 1 the optimal parameters and quality measures of all pertinent curves are indicated. In order to measure the model performance, the Aike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are calculated.

Table 1 reveals that the general skew uniform distribution provides the best fit of

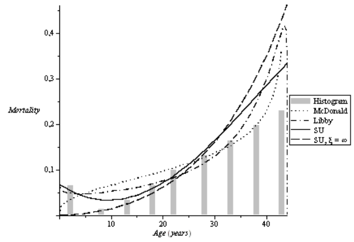


Figure 6. COVID-19 deaths per 100000 persons in the world population as a function of age.

Table 1. *Fitting COVID 19 death rates by six distributions.*

Distribution	k	Loglikelihood l	AIC= $2k-2l$	BIC= $k\ln(n)-2l$	Optimal parameter values
Beta	2	32.63	-61.26	-55.21	$a=1.407$, $b=0.6841$
McDonald's	3	32.91	-59.82	-50.75	$a=7.786$, $b=0.6837$, $p=0.1671$
Libby	3	37.66	-69.32	-60.25	$a=0.8649$, $b=1.07$, $c=0.2084$
SU with $\xi=\infty$	2	6.30	-8.6	-2.55	$a=7.538\cdot 10^5$, $b=2.589\cdot 10^5$
SU	3	38.57	-71.14	-62.07	$a=4.638$, $b=-0.6807$, $\xi=1.035$
Kumaraswamy	2	32.38	-60.76	-54.71	$a=1.428$, $b=0.6795$

k : number of parameters, sample size: $n=152$

Densities defined over the interval $[0,1]$:
McDonald's generalized beta: $f(x)=\frac{px^{ap-1}(1-x^p)^{b-1}}{B(a,b)}$, $(a,b>0)$
Libby and Novick's generalized beta: $f(x)=\frac{c^a x^{a-1}(1-x)^b}{B(a,b)(1-(1-c)x)^{a+b}}$, $(a,b,c>0)$
Kumaraswamy: $f(x)=abx^{a-1}(1-x^a)^{b-1}$, $(a,b>0)$
(Nadarajah (2005, Sections 4 and 5), Kumaraswamy (1980)).

the data. It has the maximal loglikelihood and the statistical measures AIC and BIC are minimal. In particular,the SU fits the mortality rates for low ages better than the other distributions do.

5.2. Online product reviews

Next, we fit the above distributions to the empirical data of an Amazon´s online product review of a music CD, see Fig. 7. The application makes use of the data in Hu et al. (2007, p. 11, Fig. 5b). The histogram presents the number of ratings with

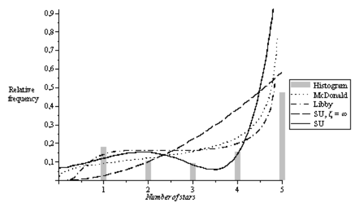


Figure 7. Product ratings of a music CD.

one to five stars. Tab. 2 gives the statistical quality measures and optimal parameters for the fitting.

Table 2. *Fitting product ratings by six distributions.*

Distribution	k	Loglikelihood l	AIC= $2k-2l$	BIC= $k\ln(n)-2l$	Optimal parameter values
Beta	2	35.67	-67.34	-62.63	$a=1.189$, $b=0.4752$
McDonald's	3	35.78	-66.56	-58.49	$a=0.6350$, $b=0.4632$, $p=2.056$
Libby	3	36.96	-67.92	-60.85	$a=7483$, $b=0.3706$, $c=20379$
SU with $\xi=\infty$	2	10.33	-16.66	-11.95	$a=1.520\cdot 10^6$, $b=51971$
SU	3	41.66	-77.32	-70.25	$a=-10.86$, $b=25.69$, $\xi=0.1167$
Kumaraswamy	2	35.71	-67.42	-62.71	$a=1.227$, $b=0.4741$

k : number of parameters, sample size: $n=78$

As in Section 5.1, the general skew uniform distribution provides the best fit of the data, and the curves of the beta distribution, its generalization of McDonald and the Kumaraswamy distribution are indistinguishable from each other in Fig. 7. Similar to the previous application, the SU fits the frequencies of 3-star and 4-star evaluations much better than the other distributions do.

6. Conclusions

We study a new class of skew uniform distributions which arises by applying a procedure of Cortés et al. (2018) to the uniform distribution. We investigate the density and the cumulative distribution function and study several characteristics like moments, shape, quantile function and Hazard rate function. The Shannon entropy and the maximum likelihood equations are also developed. The new probability distribution is applied to model COVID-19 death rates and online product reviews.

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